M.R.P. 1046
S.C. 111/232
3rd April, 1957

An approximate relation between slant visibility and horizontal visibility at ground level

by

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METEOROLOGICAL
OFFICE
16 SEP 1957
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Preface

A method for calculating the variation of visibility with height in a fog was suggested some years ago (Stewart, 1955, App. VI). It was not extended to give estimates of slant visibility at that time because it was hoped on the one hand that observations on the composition of fog would soon be available to verify the model on which the calculations were based, and on the other hand that direct observations of slant visibility would be made on a fairly wide scale and would make indirect estimates unnecessary. Neither of these things has happened, and in view of the continuing interest in problems of slant visibility it has seemed worth while to extend the previous calculations to give values of slant visibility (instead of horizontal visibility at different levels) and at the same time to test the theoretical values as fully as possible against existing data. The data used for testing are observations of slant visibility at Cardington and of vertical visibility at London Airport. It is shown that both sets of observations can be fitted, within broad limits, by a single curve, which represents the best estimate of slant visibility that can be made from surface observations at the present time.

1. Introduction

The temperature gradient in a well developed fog is usually found to be close to the saturated adiabatic value (Dines 1930, Stewart 1955, Urfer 1956), as would be expected from the fact that fogs lose heat by long wave radiation from their upper surfaces and gain heat from the ground surface; the heat is presumably transferred upwards by convection currents. If it is assumed that the water condensed in rising air currents is all deposited on existing droplets and also, for simplicity, that all the droplets at a given level are of the same size, then the way in which opacity varies with height can readily be calculated. For most practical purposes the opacity at a given height is not of great interest, but it is not difficult to extend the calculations to give the more interesting quantity "slant visibility".

2. Definition of slant visibility

The visual range of an object to a given observer depends on illumination, contrast and eyesight as well as on the state of the atmosphere, but it is convenient to define "visibility", V, as the range of a standard object to a standard observer in standard conditions of illumination. If the standards are suitably specified, the visibility becomes a function only of the atmospheric attenuation coefficient for visible light, σ, given by Koschmieder's equation, $V\sigma = 3.91$, if the fog is uniform, or its extension, $\int_0^V \sigma \, dx = 3.91$, if σ varies along the line of sight. This definition can, of course, be applied to

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slant visibility from a height h (i.e., the visibility along a path from this height to the ground) as well as to horizontal visibility, and it will be used here. It should be noted, however, that the correspondence between visibility and visual range may be much less close for slant observations than it is for normal horizontal ones, because the conditions of observation are more likely to depart widely from the standard conditions used in the definition of visibility.

3. Relation between slant visibility and attenuation coefficient

The slant visibility, S, from height h_1 is given by

$$\int_0^S \sigma \, dx = 3.91$$

where the path of integration (the line of sight) extends from h_1 to ground level at an angle with the horizontal whose sine is h_1/S. If we assume that σ depends only on height, h, i.e., that the fog is uniform in the horizontal, we can rewrite the equation as

$$\frac{S}{h_1} \int_0^{h_1} \sigma \, dh = 3.91 \qquad (1)$$

or, approximately,

$$S = 4h_1 \Big/ \int_0^{h_1} \sigma \, dh \qquad (2)$$

4. Variation of attenuation coefficient with height

The attenuation coefficient of a cloud containing n drops per unit volume, of radius r, is

$$\sigma = K n \pi r^2 \qquad (3)$$

$$\text{or} \quad \sigma = K \left(\frac{9\pi}{16}\right)^{\frac{1}{3}} n^{\frac{1}{3}} m^{\frac{2}{3}} \qquad (4)$$

where m is the mass of liquid water (of unit density) per unit volume and K is the "scattering area ratio" which, for drops larger than a few microns, can be taken as equal to 2.0 (Middleton, 1952).

The attenuation coefficient at height h is therefore

$$\sigma = \sigma_0 \left(1 + \frac{\delta m}{m_0}\right)^{\frac{2}{3}} \qquad (5)$$

where σ_0 is the coefficient at ground level, m_0 the liquid water content and δm the extra water condensed by adiabatic ascent to height h, which we may write as

$$\delta m = \Gamma_w \frac{dw}{dT} h$$

where Γ_w is the saturated adiabatic lapse rate and $\frac{dw}{dT}$ the rate of change of vapour density with temperature. σ_0 can be expressed in terms of the horizontal visibility at ground level, H, by the equation $\sigma_0 = 3.91/H$, which will be taken, exactly enough, as $\sigma_0 = 4/H$; m_0 can then be obtained from (4).

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We can therefore write (5) as

$$\sigma = \frac{4}{H}\left(1 + \frac{\Gamma_w \frac{dw}{dT} \cdot h}{\frac{8}{H^{\frac{3}{2}}}\left(\frac{2}{9\pi n}\right)^{\frac{1}{2}}}\right)^{\frac{2}{3}}$$

$$= \frac{4}{H}\left(1 + A H^{\frac{3}{2}} h\right)^{\frac{2}{3}} \qquad (6)$$

where $A = \sqrt{\frac{9\pi}{128}}\sqrt{n} \cdot \Gamma_w \frac{dw}{dT}$

5. Variation of slant visibility with height

The slant visibility can be obtained by integrating (6) with respect to h and substituting in (2), which gives

$$\frac{S}{H} = \frac{5}{3} \frac{A H^{\frac{3}{2}} h_1}{\left(1 + A H^{\frac{3}{2}} h_1\right)^{5/3} - 1} \qquad (7)$$

We have thus obtained an expression for the slant visibility, S, in terms of the horizontal visibility at ground level, H, the height of observation, h_1, and a constant A which depends on temperature and on the number of fog drops per unit volume. The expression indicates, of course, that slant visibility is always less than horizontal and that its relative variation with height is greater the greater the horizontal visibility. It will be valid only when all observations are made within a large-drop water-fog; it cannot be used when the horizontal visibility at ground level, H, depends on "smoke" or "nuclei" rather than on water droplets (i.e., roughly speaking, when H is greater than about 800 yards or when relative humidity is appreciably below 100 per cent), nor does it give directly the slant visibility from heights above a fog layer.

6. Testing of the relation

Equation (7) is based on assumptions about the nature and behaviour of fogs which are probably not exactly fulfilled in practice. In the absence of reliable measurements of drop size and concentration at different levels in fog, it is impossible to test the assumptions directly. Direct comparison of equation (7) with visibility observations is also impossible, because the constant A is unknown. We can, however, draw curves of S/H as a function of $h.H^{3/2}$ for various values of A. Observations on any one occasion should lie on one of these curves and to the extent that observations on different occasions lie on a common curve, we can say that A has the same value in all fogs.

Two sets of observations and curves for two values of A have been plotted in Fig. 1. The first set of observations contains measurements of the slant visual range of a small lamp made at Cardington in 1953 and briefly reported by Stewart (1955). The visual ranges have been converted into "equivalent daylight visibilities" by the methods described in the "Observers Handbook" and should therefore correspond fairly closely to the slant visibility as defined in para. 2. Horizontal visibility was measured simultaneously by a photoelectric meter or by visual estimation. It ranged from 30 to 250 yards in the observations plotted, while slant measurements were made up to the height at which the light disappeared, from 300-500 ft.

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The second set of observations contains daylight observations of "vertical visibility" made at London Airport by noting the time for a small balloon, filled to ascend at a known rate, to disappear from view. It was assumed that the height at which the balloon disappeared was equal to the slant visibility from that height, so that each balloon ascent gave a single measurement of slant visibility. In these observations the standard conditions for visibility observations were reasonably well fulfilled, though appreciable errors may have arisen from the small size and pale colour of the balloon and from its drifting in the wind.

Fig. 1 shows that within a rather wide margin of error the majority of observations, both at Cardington and London Airport, lie on the curve with $A = 1.15 \times 10^{-6} (ft)^{-5/2}$ (corresponding to 300 droplets/c.c. at 40°F). The fit for the Cardington observations is not greatly improved if A is varied to give the best fit for each occasion separately.

The departures of the observed points from the mean curve may be due to a variety of causes, other than over-simplification of the model on which the theoretical curve is based. The observations themselves are not very accurate; at Cardington, errors of 20 per cent in both slant and horizontal visibility are quite possible and at London Airport even greater errors may be expected in vertical visibility. (Vertical visibility is, in any case, usually only reported in hundreds of feet, rounding off to the lower hundred; in these cases the observations have been plotted as if they were fifty feet higher than the reported values.) Points above the mean curve may represent observations made above the fog top, in the early stages of a fog before convective equilibrium has been established, in fogs uniformly polluted by smoke or in fogs where, for some reason, the number of droplets is unusually small. Points below the curve may represent conditions of low cloud or "lifted fog" with visibility at the ground restricted by smoke rather than water drops; fogs with smoke concentration increasing upwards; fogs in which the size of droplets near the ground is unusually small, or fogs with unusually large numbers of droplets.

It is clear that equation (7) cannot be used to find accurate values of S on individual occasions, but it probably does give the best available representation of the average behaviour of the type of fog considered - certainly much better than the assumption that fogs are uniform with height, equivalent to S/H = 1. It would, no doubt, be possible to improve the fit between theory and observation by introducing further variables such as the spread of the drop size distribution or the degree of convection in the fog, but there seems little point in doing so unless these variables can be measured.

7. Estimation of slant visibility

Equation (7) and fig. 1 do not give estimates of S in the most convenient form. Fig. 2 gives values of S at different heights, in terms of H, as derived from equation (7) with $A = 1.15 \times 10^{-6} (ft)^{-5/2}$. It represents the best estimate of slant visibility that can be made at present from surface observations alone and it is hoped that it may be of some value in connection with aircraft landing problems. As a guide to the accuracy that may be expected, the Cardington observations for h = 300 ft have been plotted on fig. 2.

Fig. 2 should be used with the following limitations in mind:

(a) It gives the most likely value of slant visibility, on average, but quite large differences may occur on some occasions.

(b) It gives visibility not visual range; by day the visual range is likely to be less than the visibility, because of poor contrast and uneven lighting; by night the range of the usual bright approach and runway lights may be several times the "visibility". Visual range can easily be calculated from visibility, if the necessary data are available.

(c) It cannot be used when H is seriously affected by smoke.

(d) It applies only to "mature" fogs, in which convection is active; as a

rough guide, it should apply to fogs with "sky invisible" but not to those with "sky visible".

(e) It does not apply to heights above the fog top, though if the height of fog top is known calculations to cover this case can be made.

(f) It has been plotted using values of constants found appropriate at Cardington and London Airport. The curves may require modification for use at other stations, though they are likely to be roughly correct for all low-level inland stations in SE. England.

8. References

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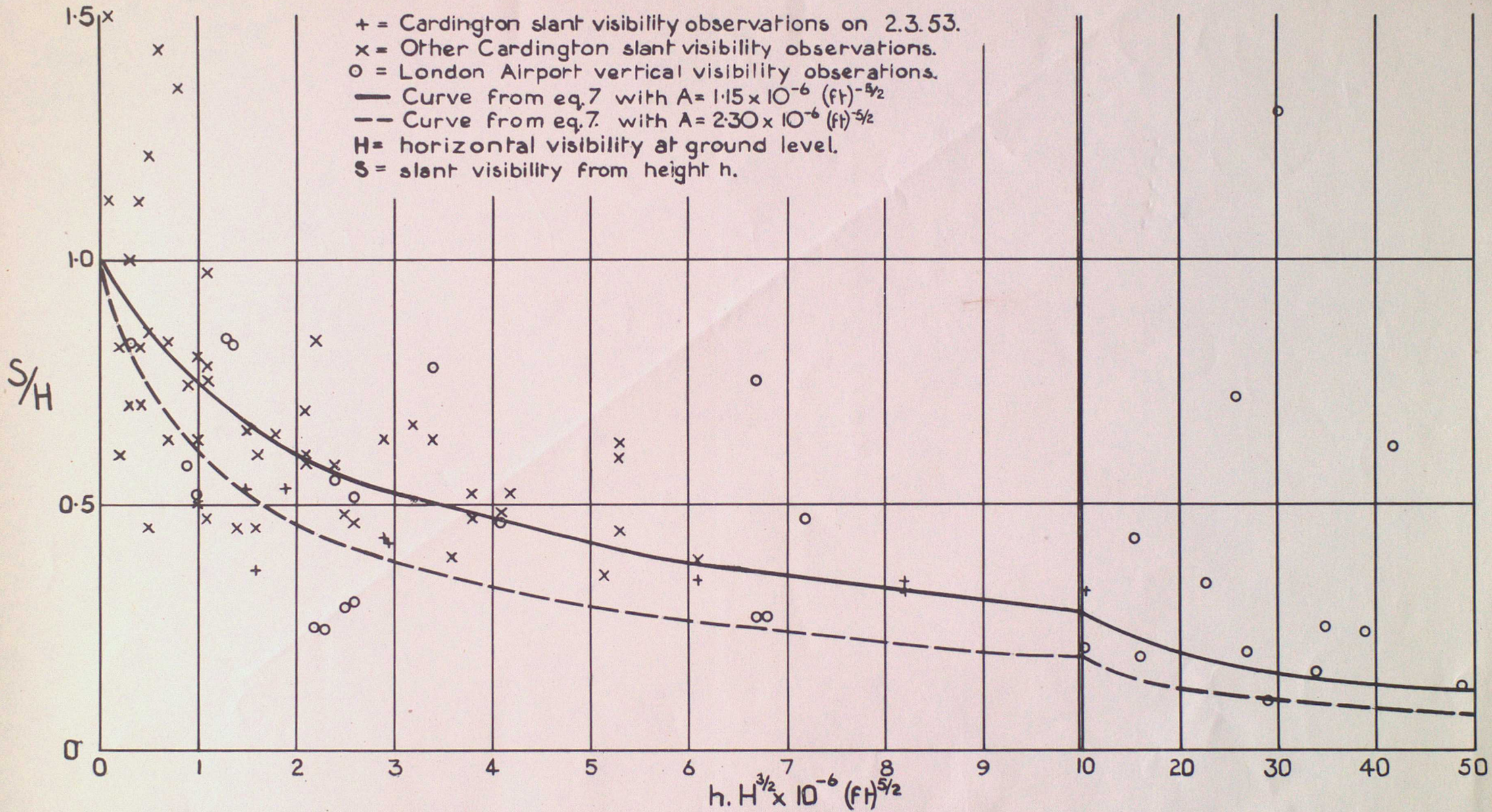

Fig 1. Relation between slant and horizontal visibility.

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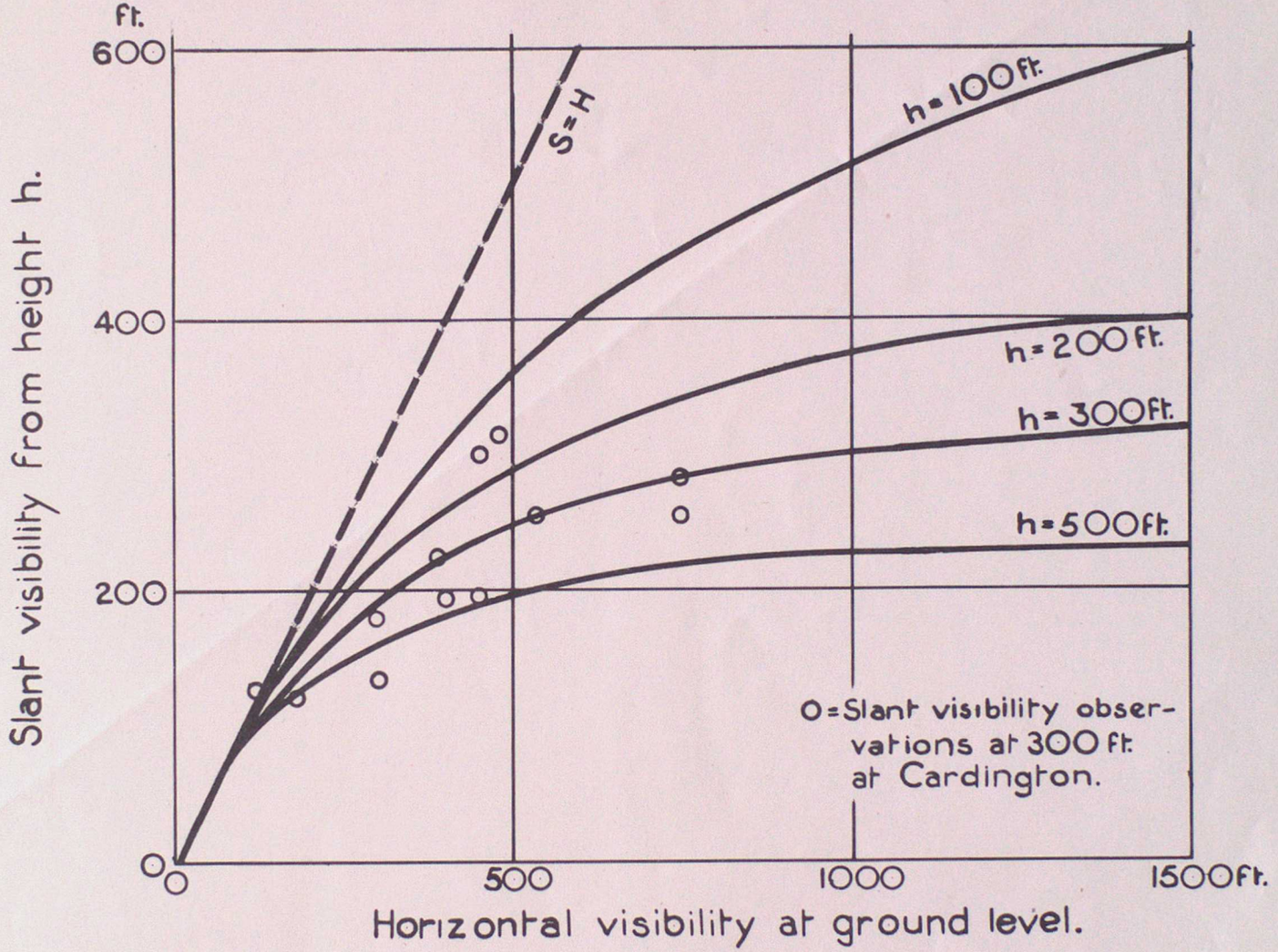

Fig. 2. Theoretical relation between slant visibility from height h and horizontal visibility at ground level.